Magnetic-Flux Penetration and Critical Currents in Superconducting Strips with Slits

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We theoretically investigate transport-current-induced magnetic-flux penetration into superconducting strip lines with slits. Even when the individual strips have no bulk pinning, geometrical barriers prevent penetration of magnetic flux into the innermost strips while flux quasistatically penetrates into the outermost slits. The critical current of strip lines with $2N$ slits at zero applied magnetic field is found to be enhanced by a factor of $(N + 1)^{1/2}$ above that of a single strip line without slits. Under suitable conditions, a domelike flux distribution due to the geometrical barrier can appear in the individual strips even in the absence of an applied magnetic field.

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Recent experimental and theoretical studies have shown that a potential barrier of geometric origin can prevent magnetic flux from penetrating into a type-II superconducting flat strip subjected to a perpendicular magnetic field with a transport current [1–3]. This geometrical-barrier effect results in a domelike distribution of magnetic field [1,2], hysteretic magnetization [1,2], and a nonzero critical current [3], despite the absence of bulk pinning. The surrounding magnetic environment also has been shown to strongly affect the geometrical barrier and to enhance the critical current of the strip [4]. In this Letter, we demonstrate how magnetic flux penetrates into a *finite* number of coplanar strips (i.e., a strip line with multiple slits) carrying a transport current. We propose a simple and effective method to enhance critical currents in strip lines without bulk pinning: Make narrow slits near the edges of the strip line, and the critical current will become larger by a factor of $(N + 1)^{1/2}$ for $2N$ slits.

First we briefly review how to calculate the critical current in a single strip without bulk pinning [3,5]. The superconducting strip under consideration has a rectangular cross section of width $2a$ and thickness d, and is infinitely extended along the z axis (i.e., the cross section occupies the area $|x| < a$ and $|y| < d/2 \ll a$). It is convenient to express the two-dimensional field distribution as an analytic function $H(\zeta) \equiv H_y(x, y) + iH_x(x, y)$ of the complex variable $\zeta \equiv x + iy$ [1,5]. When the strip carries a transport current I_t along the z axis in the absence of an applied magnetic field, the complex field around the strip in the Meissner state is [6,7] $H(\zeta) = (I_t/2\pi)(\zeta^2 - a^2)^{-1/2}$. The magnetic field at the edge at $x = a + 0$ and $y = 0$ is obtained as $H(a + \delta) = H_y(x = a + \delta, y = 0) \simeq (I_t/2\pi)(2a\delta)^{-1/2}$, where we have introduced a cutoff length δ on the order of the thickness d [1–3]. The critical current I_{cs} for the strip without bulk pinning is given by the current at which the edge field $H(a + \delta)$ reaches a certain flux-entry field H_s [2,3]. The field H_s may be equal to the lower critical field H_{c1} in the absence of a Bean-Livingston barrier [8] or may be on the order of the thermodynamic field H_c in the presence of an ideal surface barrier [9]. Thermal fluctuations, however, may cause the effective H_s to be smaller than that without thermally activated vortex nucleation. We thus find the critical current of a single strip at zero magnetic field to be [3]

$$I_{cs} = 2\pi H_s(2a\delta)^{1/2}. \tag{1}$$

The magnetic-field distribution around multiple strips is much more complicated than that around a single strip. It is possible to investigate flux penetration into a *periodic* array of an *infinite* number of strips by using a simple transformation technique [10]. In the present paper, however, we show how to calculate the behavior for a strip line consisting of a *finite* number of strips. We first consider a strip line of total width $2a$ with slits at $c < x < b$ and $-b < x < -c$, where $a > b > c$. In other words, the strip line consists of three individual coplanar strips, as shown in Fig. 1. The strip thickness d is assumed to be somewhat larger than the penetration depth λ, but much smaller than the smallest of $a - b$, $b - c$, and $2c$. We consider flux penetration into a strip line carrying transport current I_t along the z axis in the absence of an applied magnetic field. The three individual strips are infinitely long along the z axis, and their ends are connected at $z \to \pm\infty$. The total current I_t is therefore divided among the three strips: the inner strip at $|x| < c$ carries I_{in} and the two outer strips at $b < |x| < a$ carry I_{out} each, where $I_t = I_{\text{in}} + 2I_{\text{out}}$.

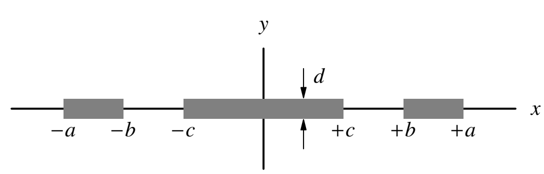

FIG. 1. Superconducting strip line with slits. Superconducting strips (thickness d, $|y| < d/2$, infinitely extended along the z axis) occupy the gray areas: the inner strip at $|x| \le c$, the outer strips at $b < |x| < a$, and the slits at $c < |x| < b$. The inner strip carries a net current I_{in}, the two outer strips carry I_{out} each, and the total transport current is $I_t = I_{\text{in}} + 2I_{\text{out}}$.

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The complex field for such a strip line, provided that all the strips remain in the Meissner state, is obtained by the conformal mapping

$$H(\zeta) = \frac{I_t}{2\pi} \frac{\zeta^2 - \gamma^2}{[(\zeta^2 - a^2)(\zeta^2 - b^2)(\zeta^2 - c^2)]^{1/2}}, \quad (2)$$

where the parameter γ ($c < \gamma < b$) depends on I_t. Under suitable conditions, however, domelike distributions of magnetic flux due to the geometrical barrier [1–3] can occur in the outer strips. The corresponding complex field is then given by

$$H(\zeta) = \frac{I_t}{2\pi}\left[\frac{(\zeta^2 - \alpha^2)(\zeta^2 - \beta^2)}{(\zeta^2 - a^2)(\zeta^2 - b^2)(\zeta^2 - c^2)}\right]^{1/2}, \quad (3)$$

where the domelike flux distributions are at $\beta < |x| < \alpha$ in the outer strips for $b < \beta < \alpha < a$.

With increasing I_t, the flux-penetration process proceeds in three steps: (i) no flux penetration, (ii) quasistatic penetration, and (iii) continuous penetration, producing a resistive state. In addition, two kinds of step (ii) exist: (ii-a) *without* domelike flux distributions and (ii-b) *with* domelike flux distributions in the outer strips. See Fig. 2.

Step (i), $0 < I_t < I_1$: Magnetic flux cannot penetrate into the strips, and the field distribution is given by Eq. (2). The parameter $\gamma = \gamma_1$ ($c < \gamma_1 < b$) in Eq. (2) is determined by the condition that the total magnetic flux in the slits is zero, $\int_b^c dx\, H_y(x, y = 0) = 0$. The resulting expression for γ_1 is constant, independent of I_t:

$$\gamma_1 = c[\Pi(-1 + c^2/b^2, k)/K(k)]^{1/2}, \quad (4)$$

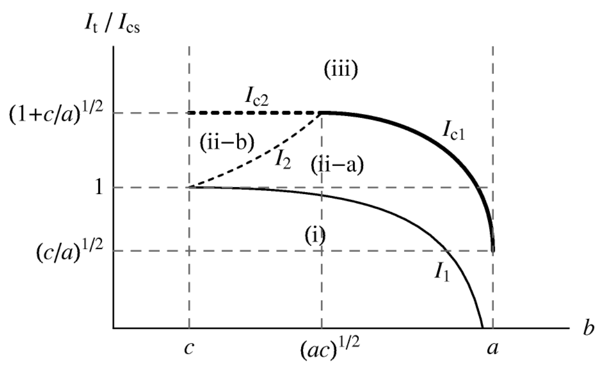

FIG. 2. Characteristic currents in the I_t versus b plane for magnetic-flux penetration into the strip line. The characteristic currents I_1 (thin solid), I_2 (thin dashed), I_{c1} (bold solid), and I_{c2} (bold dashed) are given by Eqs. (5), (8), (7), and (10), respectively. Region (i), $0 < I_t < I_1$: no magnetic flux penetrates into the strips. Region (ii-a), $I_1 < I_t < (I_2 \text{ or } I_{c1})$ [i.e., $I_1 < I_t < I_2$ for $b < (ac)^{1/2}$ and $I_1 < I_t < I_{c1}$ for $b > (ac)^{1/2}$]: magnetic flux penetrates into slits *without* domelike flux distributions. Region (ii-b), $I_2 < I_t < I_{c2}$: magnetic flux penetrates into slits *with* domelike flux distributions in the outer strips. Region (iii), $I_t > I_c$ [where the critical current I_c is given by $I_c = I_{c2}$ for $b < (ac)^{1/2}$ and $I_c = I_{c1}$ for $b > (ac)^{1/2}$]: flux continuously penetrates the strips and annihilates at the center, producing a resistive state.

where $K(k)$ and $\Pi(p, k)$ are the complete elliptic integrals of the first and third kind, respectively, and $k = (a/b)[(b^2 - c^2)/(a^2 - c^2)]^{1/2}$. The magnetic fields at the edges of the strips are obtained from Eq. (2) as $H_{e,a} = H(a + \delta)$, $H_{e,b} = H(b - \delta)$, and $H_{e,c} = H(c + \delta)$, where $\delta \sim d$. The edge fields increase linearly with increasing current $I_t > 0$, and the inequalities $H_{e,a} \geq H_{e,c} > |H_{e,b}|$ hold in most cases. The magnetic fields at the edges are smaller than the flux-entry field H_s for $0 < I_t < I_1$; i.e., $\max(H_{e,a}, |H_{e,b}|, H_{e,c}) < H_s$, and no magnetic flux penetrates into the strips. However, at $I_t = I_1$, $H_{e,a}$ attains the value H_s, and magnetic flux nucleates at the outermost edges, $x = \pm a$. The upper limit of I_t for step (i), obtained from Eq. (2), is given by

$$\frac{I_1}{I_{cs}} = \frac{[(a^2 - b^2)(a^2 - c^2)]^{1/2}}{a^2 - \gamma_1^2}, \quad (5)$$

where I_{cs} is given by Eq. (1). The value of I_1 is always less than I_{cs}, as shown by the thin solid curve in Fig. 2. The field and current distributions at I_1 are shown by the thin solid curves in Fig. 3.

Step (ii-a), $I_1 < I_t < (I_2 \text{ or } I_{c1})$: Magnetic flux quasistatically penetrates into the slits *without* domelike-flux distributions in the strips. Here magnetic flux nucleates at the outermost edges $x = \pm a$, flows entirely across the outer strips, and enters into the slits. The edge field $H_{e,a} = H_s$ remains constant, and magnetic flux penetrates only so

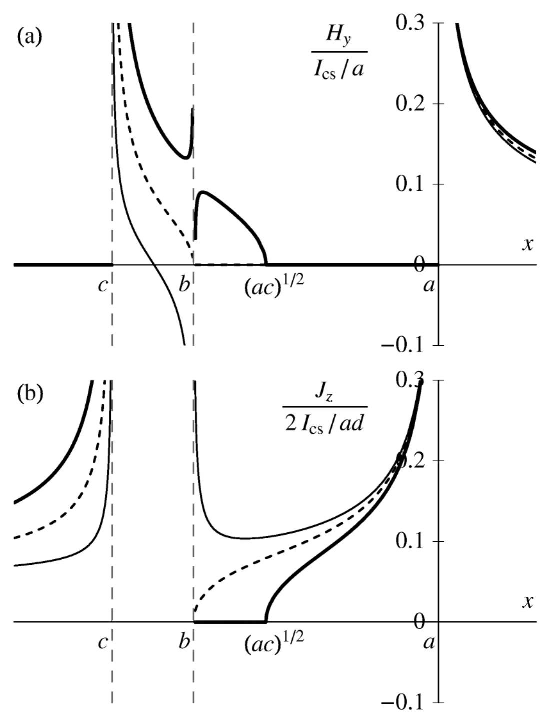

FIG. 3. Distributions of (a) the magnetic field $H_y(x, 0) = \mathrm{Re}[H(x)]$ and (b) the current density $J_z(x) = (2/d)\,\mathrm{Im}[H(x)]$ at $y = 0$ for $I_t = I_1$ (thin solid), $I_t = I_2$ (dashed), and $I_t = I_{c2}$ (bold solid). The distributions are calculated from Eqs. (2) and (3) with $b/a = 0.85$ and $c/a = 0.8$.

long as I_t increases. The induced electric field depends on the ramp rate of the transport current, dI_t/dt, but is negligibly small for quasistatic ramping, $dI_t/dt \to 0$. Such a flux-penetration process is similar to that in the critical-state model [11] because of flux pinning at the slits. The field distribution is now given by Eq. (2) in which the parameter $\gamma = \gamma_2(I_t)$ is an increasing function of I_t. The condition that $H_{e,a} = H_s$ yields

$$\gamma_2(I_t) = \left\{a^2 - \frac{I_{cs}}{I_t}[(a^2 - b^2)(a^2 - c^2)]^{1/2}\right\}^{1/2}. \quad (6)$$

There exist two characteristic currents as upper limits of the step (ii-a), the critical current I_{c1} for $b > (ac)^{1/2}$ and I_2 for $b < (ac)^{1/2}$.

For $b > (ac)^{1/2}$, $H_{e,c}$ reaches the flux-entry field H_s at $I_t = I_{c1}$ while $c < \gamma_2 < b$ holds. The critical current I_{c1} is determined by $H_{e,a} = H_{e,c} = H_s$ from Eq. (2):

$$\frac{I_{c1}}{I_{cs}} = \frac{[a(a^2 - b^2)]^{1/2} + [c(b^2 - c^2)]^{1/2}}{[a(a^2 - c^2)]^{1/2}}. \quad (7)$$

As shown as the bold solid curve in Fig. 2, the right-hand side of Eq. (7) achieves its maximum value $(1 + c/a)^{1/2}$ at $b = (ac)^{1/2}$, and decreases monotonically for $(ac)^{1/2} < b < a$.

For $c < b < (ac)^{1/2}$, on the other hand, the step (ii-a) for $I_1 < I_t < I_2$ terminates when $\gamma_2(I_t) = b$ at $I_t = I_2$, while $H_{e,c} < H_s$ holds. The value of I_2 is determined by $\gamma_2(I_2) = b$ in Eq. (6),

$$\frac{I_2}{I_{cs}} = \left(\frac{a^2 - c^2}{a^2 - b^2}\right)^{1/2}, \quad (8)$$

which is larger than 1, as shown as the thin dashed curve in Fig. 2. The field and current distributions at I_2 are shown by the dashed curves in Fig. 3.

Step (ii-b), $I_2 < I_t < I_{c2}$, $c < b < (ac)^{1/2}$: Magnetic flux quasistatically penetrates into the slits *with* domelike-flux distributions in the outer strips. For currents I_t just above I_2, nucleating vortices are no longer swept entirely across the outermost strips into the slits as in step (ii-a). Instead, some vortices remain in the outer strips. Hence the Meissner-state Eq. (2) is no longer valid, and the correct expression of the complex field for step (ii-b) is now given by Eq. (3). One edge of the domelike distribution of magnetic flux, β in Eq. (3), should be close to b; in other words, $\beta - b$ is of the order of the thickness d. Some of the magnetic flux nucleated at the outermost edges $x = \pm a$ remains in the domelike distributions at $b \simeq \beta < |x| < \alpha$ in the outer strips, and the rest exits from $x = \pm b$ and enters into the slits. The outer edge of the domelike distributions of magnetic flux, $\alpha(I_t)$, is determined from Eq. (3) by $H_{e,a} = H_s$ with $\beta \simeq b$, and is given by

$$\alpha(I_t) = [a^2 - (I_{cs}/I_t)^2(a^2 - c^2)]^{1/2}. \quad (9)$$

The critical current I_{c2} for $c < b < (ac)^{1/2}$ is determined from Eq. (3) by $H_{e,a} = H_{e,c} = H_s$,

$$I_{c2}/I_{cs} = (1 + c/a)^{1/2}, \quad (10)$$

which is shown as the bold dashed line in Fig. 2. Note that when narrow slits are present close to the edges, such that $a - c \ll a$, the critical current I_{c2} can be larger than I_{cs} by the factor of $(1 + c/a)^{1/2} \simeq \sqrt{2}$.

Figure 3 shows the distributions of the magnetic field H_y and the current density J_z at $y = 0$ for $b < (ac)^{1/2}$ at several currents I_t in step (ii). At the end of step (ii-b), at $I_t = I_{c2}$, a domelike distribution of H_y exists in the region $b < x < \alpha(I_{c2}) = (ac)^{1/2}$. The values of H_y and J_z near the outermost edge $x \sim a$ are almost unchanged. Note that the J_z at $b < x < a$ (and hence the net current I_{out} in each outer strip) *decreases* with increasing $I_t = I_{\text{in}} + 2I_{\text{out}}$, whereas the J_z at $0 < x < c$ (and hence the net current I_{in} in the inner strip) *increases*. The redistribution of the currents between I_{in} and I_{out} weakens the concentration of the current near the outermost edges, and is responsible for the critical-current enhancement.

Step (iii), $I_t > I_c$: Magnetic flux continuously penetrates into the strips and flows toward the center of the strip line, $x = 0$, where positive flux from $x > 0$ and negative flux from $x < 0$ annihilate. A nonzero steady-state electric field occurs when I_t exceeds the critical current, $I_c = I_{c1}$ in Eq. (7) for $b > (ac)^{1/2}$ or $I_c = I_{c2}$ in Eq. (10) for $b < (ac)^{1/2}$. See Fig. 2.

We have extended the above approach to the case of $2N + 1$ coplanar strips (i.e., a symmetric strip line with $2N$ slits), where $N \geq 1$. The superconducting strips occupy the regions $b_n < |x| < a_n$ for $0 \leq n \leq N$ with $0 = b_0 < a_0 < b_1 < a_1 < \cdots < b_{N-1} < a_{N-1} < b_N < a_N$. With regard to the parameters a_n, we introduce an equation with respect to s,

$$\sum_{n=0}^{N} \frac{a_n}{s^2 - a_n^2} = 0, \quad (11)$$

which has N positive solutions $s = s_n$ in the range of $a_{n-1} < s_n < a_n$ for $1 \leq n \leq N$. Equation (11) with $N = 1$, for example, has a positive solution of $s_1 = (a_0 a_1)^{1/2}$. If the configuration of strips satisfies $b_n \geq s_n$ for all $1 \leq n \leq N$, static magnetic flux cannot exist in any of the strips. The complex field for $2N + 1$ strips containing no magnetic flux is given by [cf. Eq. (2)]

$$H(\zeta) = \frac{I_t}{2\pi} \frac{\prod_{n=1}^{N}(\zeta^2 - \gamma_n^2)}{[(\zeta^2 - a_0^2)\prod_{n=1}^{N}(\zeta^2 - a_n^2)(\zeta^2 - b_n^2)]^{1/2}}. \quad (12)$$

The critical flux-entry condition for all strips, coupled equations $H(a_n + \delta) = H_s$ using Eq. (12) for $0 \leq n \leq N$ at $I_t = I_{c1,N}$, leads to the critical current [cf. Eq. (7)],

$$\frac{I_{c1,N}}{I_{cs}} = \sum_{k=0}^{N} \left[\frac{a_k}{a_N} \frac{\prod_{n=1}^{N}(a_k^2 - b_n^2)}{\prod_{n=0,n\neq k}^{N}(a_k^2 - a_n^2)} \right]^{1/2}, \qquad (13)$$

where $\prod_{n=0,n\neq k}^{N}$ means the product over $0 \leq n \leq N$ except for the factor with $n = k$.

If $b_n < s_n$ holds for all $1 \leq n \leq N$, on the other hand, domelike distributions of magnetic flux can exist in all strips except for the innermost one at $|x| < a_0$. The corresponding complex field is given by [cf. Eq. (3)]

$$H(\zeta) = \frac{I_t}{2\pi} \left[\frac{1}{\zeta^2 - a_0^2} \prod_{n=1}^{N} \frac{(\zeta^2 - \alpha_n^2)(\zeta^2 - \beta_n^2)}{(\zeta^2 - a_n^2)(\zeta^2 - b_n^2)} \right]^{1/2}, \qquad (14)$$

where domelike distributions of magnetic flux are at $b_n \simeq \beta_n < |x| < \alpha_n$ for $1 \leq n \leq N$. (See also Ref. [12].) The critical flux-entry condition for all strips, $H(a_n + \delta) = H_s$ using Eq. (14) for $0 \leq n \leq N$ at $I_t = I_{c2,N}$, leads to the critical current $I_{c2,N}$ [cf. Eq. (10)],

$$\frac{I_{c2,N}}{I_{cs}} = \left(\sum_{k=0}^{N} \frac{a_k}{a_N} \right)^{1/2}. \qquad (15)$$

The critical currents in Eqs. (13) and (15) obey the inequality $I_{c1,N} \leq I_{c2,N}$; in other words, $I_{c2,N}$ is the maximized value of $I_{c1,N}$ with changing b_n for fixed a_n. The $I_{c1,N}$ coincide with $I_{c2,N}$ when $b_n = s_n$ holds for all $1 \leq n \leq N$. The $I_{c2,N}$ is further maximized as $I_{c2,N}/I_{cs} \simeq (N + 1)^{1/2}$, when all slits are narrow and are close to the outermost edges, i.e., when $a_N - a_0 \ll a_N$. The critical-current enhancement arises from the flux-pinning effect of the slits and is similar to flux pinning in superconducting layers [9].

In summary, we have investigated the penetration of magnetic flux into current-carrying strip lines with slits in the absence of an applied magnetic field. Domelike distributions of magnetic flux due to the geometrical barrier can exist in strips even without an applied magnetic field. The slits act as pinning centers, and penetration of magnetic flux is delayed. The critical current I_c of a strip line with slits is larger than that of a single strip line without slits, I_{cs}. For $2N + 1$ coplanar strips (i.e., strip lines with $2N$ slits), the critical current can be as high as $I_c/I_{cs} \simeq (N + 1)^{1/2}$ when the slits are narrow and lie close to the outermost edges.

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[12] The general expression for the complex field for an arbitrary number of coplanar strips is given as a product of contributions from each strip. The complex field for N' strips exposed to an applied field H_a is given by

$$H(\zeta) = H_a \left[\prod_{n=1}^{N'} \frac{(\zeta - \alpha_n')(\zeta - \beta_n')}{(\zeta - a_n')(\zeta - b_n')} \right]^{1/2}, \qquad (16)$$

where the nth strip is at $a_n' < x < b_n'$ and domelike flux distributions are at $\alpha_n' < x < \beta_n'$. Equation (16) with $\alpha_n' = \beta_n'$ for all n corresponds to the complex field in the Meissner state.